Intuition Test: Correlations between Random Events

Mik Zlatin

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Intro

These puzzles are meant to test your intuition for correlations between random events, and to provide at least one example of a situation where an intuitive argument leads to an incorrect result.

Puzzles

2.1 A die game

Alice and Bob are playing dice. Alice secretly rolls a fair die until she gets a 2.

Bob says: "The expected number of rolls you made is 6."

Alice says: "Actually, I only rolled even numbers. Does that change your belief?"

Bob ponders this for a minute. Then says: "Yes! Given the information that you only rolled even numbers, the expected number of rolls you made should be 3."

Question: Is Bob correct?

If not, is the true expected number more or less than 3?

2.2 Random graphs

A graph G on n vertices is sampled by including each edge independently with probability p, and then put in a box out of eyesight.

You find yourself idly wondering whether G is 3-colorable. Suddenly, a truth-telling genie appears and informs you that G has a Hamiltonian cycle.

Question: Does this influence your probability that G is 3-colorable?

Is it more likely, less likely, or the same?

2.3   Partially ordered sets

Let P be a partially ordered set (poset) on n elements. A linear extension of P is a total order on n which is consistent with the partial order P.

We will pick a linear extension of P, uniformly at random.

Now fix three elements x, y and z which are incomparable in P. Let A be the event that, in the linear extension, we have $x < y$. Similarly, let B be the event that $x < z$ in the linear extension.

Question 1: Are A and B positively correlated?

Does your answer hold for any poset?

Follow up question! Now suppose we fix four elements w, x, y, z which are incomparable in P.

Let C be the event that $z < x < w$ in the linear extension of P. Let D be the event that $z < y < w$ in the linear extension of P.

Question 2: Are C and D positively correlated?

There is an intuitive argument that both of these pairs of random events should be positively correlated. However, I will give you one hint: For one of the two questions above, the answer is "yes". For the other, the answer is "not necessarily".

Can you tell which is which?